The Effect of Peer Selection with Hopcount or Delay Constraint on Peer-to-Peer File Distribution

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Abstract—To ensure efficient content retrieval in a distributed network (e.g. peer-to-peer network), a connected peer with required replica should be "close" (depending on what criteria we are using) to the requesting peer. We model the peer selection problem as finding the most nearby peer with respect to the number of hops, and the delay (latency) separately.

Both the exact probability density function of the hopcount and the asymptotic analysis of the delay indicate that a minimum peer group size exists to offer an acceptable content distribution service. The analysis are presented for a random graph with i.i.d exponential link weights, and these observations may shed light on the architectural design of a P2P storage system.

I. INTRODUCTION

It has been foreseen that the next generation of Internet will evolve to a distributed fashion (certain central control may still exist) by implementing the idea of P2P networking, where mutual benefit between end-users is possible. By distributing the storage and retrieving functionalities to peers, content is stored either as an entire copy or pieces of chunks, and should be able to be retrieved once needed. Different criteria (i.e. in bandwidth, latency, or distance) are assigned to optimize the peer selecting procedure, and we are interested in modeling the performance to get access to the most nearby peer (in hopcount and delay separately) from the requesting peer.

We confine to the following problem: given a network of size \( N \) with \( m \) randomly scattered peers possessing the requested content, what is the distribution of the hopcount and/or delay to the most nearby peer from a requesting node? The peer who initiates the downloading is called the requesting peer, and the most nearby peer is denoted as the destination peer in this paper. By examining the peer selection process, a fundamental question in the P2P network of how many replicas needs to be distributed into the network so that a good service quality can be offered when retrieving the content can be answered as discussed in Section II-A and II-B.

II. PROBLEM MODELING AND ANALYSIS

Modeling the distribution of the number of hops and/or latency towards the nearest peer among a set of peers is presented for an Erdős Rényi random graph, which describes the intermittent joining and leaving of nodes in both peer-to-peer and ad-hoc networks reasonably well. If we assign link weights that are i.i.d. exponentially distributed with mean one, the shortest path tree is precisely a uniform recursive tree (URT) as shown in [2]. A URT of size \( N \) is a random tree that starts from the root \( A \), and where at each stage a new node is attached uniformly to one of the existing nodes until the total number of nodes reaches \( N \).

A. Hopcount distribution to the nearest peer

The number of hops from the requesting peer to the most nearby peer, denoted as \( h_N(m) \) is the minimum number of hops among the set of shortest paths from the requesting device to the \( m \) peers in the network of size \( N \). In this paper, we are interested in the hopcount distribution starting from one, which excludes the event \( h_N(m) = 0 \). The conditional probability is:

\[
\Pr[h_N(m)=j|h_N(m)\neq0] = \frac{1}{1-\frac{m}{N}} \Pr[H_N(m)=j]
\]

where \( j = 1, 2, \ldots N \), \( \Pr[h_N(m)=0] = \frac{m}{N} \), and \( H_N(m) \) denotes to the event that the hopcount starts from zero \( (j = 0, 1, 2, 3, \ldots) \).

The nearest peer selection (in number of hops) is intrinsically analogous to the "hopcount to an anycast group", and the probability of \( \Pr[H_N(m)=j] \) has been treated in [1, Chapter 18] in detail.

Equation (1) can be used to estimate the number of peers needed in the network to provide a certain content delivery service. In Fig. 1, we plot \( \Pr[h_N(m)=j] \) for \( j \leq 4 \) as a function of peer fraction \( \frac{m}{N} \) in the network for different network sizes. If the operator of a CDN with 40 routers uniformly scattered 4 servers (peer fraction around 10%) into the network, he can already claim that approximately in 98% of the cases, any user request will reach a server within 4 hops \( (j \leq 4) \). While placing more servers in the network will not improve the performance significantly.

B. Weight of the shortest path to the first encountered peer

In [1, Chapter 16.2], the shortest path problem between two arbitrary nodes in the complete graph with exponential distributed link weight has been rephrased as a Markov discovery process evolving as a function of time from the source to the destination, which can also be described as a continuous time Markov chain with \( N \) states. The transition rate from state \( n \) \( (n \leq N) \) with \( n \) already discovered node to the next state is \( \lambda_{n,n+1} = n(N-n) \). The inter-attachment time \( \tau_n \) between the \( n-th \) and \( (n+1)-th \) node in the SPT for \( n=1, 2, \ldots, N-1 \) is exponentially distributed with parameter \( n(N-n) \). Again, the resulting SPT is exactly a uniform recursive tree.

The exact generating function \( \varphi_{W_N,m}(z) = E[e^{-zW_N,m}] \) of the weight \( W_N,m \) of the shortest path from an arbitrary node
to the first encountered peer among $m$ peers can be formulated as
\[
\varphi_{NW;m}(z) = \sum_{k=1}^{N-m} E[e^{-zY_m}] \Pr[Y_m(k)] = \frac{m(N-1-m)!}{(N-1)!} \sum_{k=1}^{N-m} \frac{(N-1-k)!}{(N-m-k)!} \prod_{n=1}^{k} n(n-n) 
\]
where $\Pr[Y_m(k)]$ represents the probability that the $k$-th attached node is the first encountered peer among the $m$ peers in the URT.

The inverse Laplace transform of (2) can be computed, but the result is not appealing. Computation of the probability distribution function of the scaled random variable $NW_{N;m} = \frac{\varphi_{NW;m}(z)}{\varphi_{NW;m}(1)}$ for the asymptotic regime $N \rightarrow \infty$ gives more insight [6].

\[
\lim_{N \rightarrow \infty} \Pr \left[ NW_{N;m} - \ln \frac{N}{m} \leq y \right] = e^{-my}m^{m+1}e^{me^{-y}} \int_{e^{-y}}^{\infty} e^{-u} \frac{du}{u^{m+1}} 
\]
where the asymptotic distribution function resembles a Fermi-Dirac distribution that appears in statistical physics as shown in Fig. 2. The Fermi-Dirac distribution function for any $t$ is

\[
\lim_{N \rightarrow \infty} \Pr \left[ NW_{N;m} - \ln \frac{N}{m} \leq t \right] = \frac{1}{1 + e^{-t}} 
\]

Fig. 2 illustrates that, a relatively small peer group $m \approx 5$ is sufficient to offer a good service quality (i.e. the end-to-end delay) because discovering more numbers of peers only improves the performance logarithmically, i.e. marginally. Further, the simple Fermi-Dirac distribution can be employed for estimation, because if $m > 5$, the curves are very close to the asymptotic regime.

III. CONCLUSION AND POTENTIAL INFLUENCE

The results and analysis presented in this paper provide a general instruction to the number of replicas needs to be distributed into a P2P network to provide an acceptable service quality. When the hopcount and delay is the major concern of a P2P network, the performance can be guaranteed by discovering a proper number of peers as analyzed in Section II-A and II-B, while a long peer list is not needed.

Among the numerous paradigms of peer-to-peer application in today’s Internet, our analysis can be useful in the following two regimes.

- Content distribution network: A CDN [5] consists of a collection of surrogate servers, and for each request, the CDN attempts to locate the closest server (i.e. geographically, network latency, etc.) to serve the client on behalf of the original server.

- Group discovery based P2P networking: A typical example is a content storage system, i.e. CFS [7], which provide guarantees on the efficiency and robustness for file storage and retrieval by sharing and distributing resources to a cooperative group of users. To start retrieving content, a list of servers, who are free to come and go needs to be located.

For other popular P2P applications, i.e. Bit Torrent, where the hopcount and delay may be less important, it is also worth to investigate the performance of peer selection based on bandwidth.

REFERENCES


